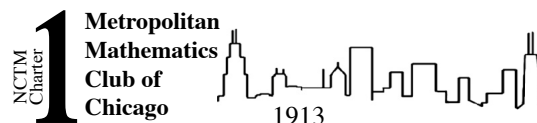


POINTS AND ANGLES

Newsletter of the Metropolitan
Mathematics Club of Chicago



Volume XXXVII

February 2003

No. 6

From Algebra to Calculus with Sketchpad 4

Nicholas Jackiw and Steve Rasmussen, Key Curriculum Press

BY SIMONETTE URBAIN

Our mathematical tour of the small screen continues as Nicholas Jackiw and Steve Rasmussen introduce us to Sketchpad 4. No longer just a tool for teaching geometry, Sketchpad 4 can be used to teach mathematical concepts from integer operations to summation of sine waves to slope fields in integral calculus.

Nicholas Jackiw is the original designer and developer of the *Geometer's Sketchpad*. He began work on the software as an undergraduate at Swarthmore College, where he received his Bachelor's degree in English Literature with a concentration in Computer Science. He has overseen the programs translation into more than a dozen languages. Nick presently serves as the Chief Technology Officer of KCP Technologies, the software affiliate of Key Curriculum Press. Nick is active in the pre-service and in-service for the professional development of teachers, conducting workshops and institutes across the country.

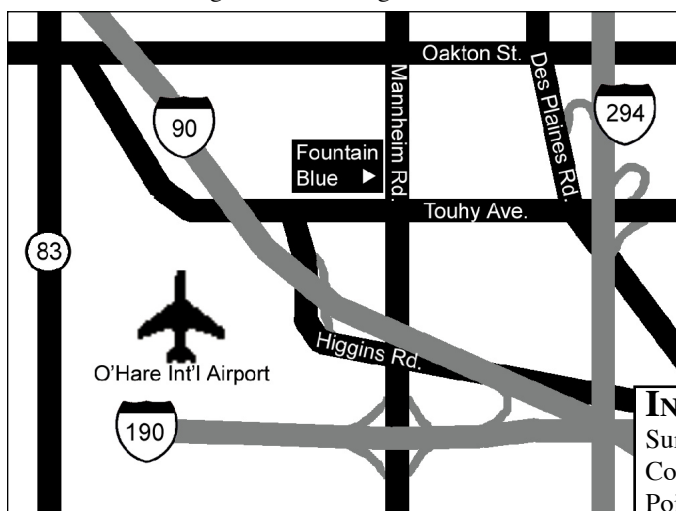
Steven Rasmussen was a leader of the *Geometer's Sketchpad* geometry software development team. He was also the editor of *Discovering Geometry: an Inductive Approach*. In 1971 Steve co-founded Key Curriculum Press and since 1988 he has been the president of the company. Steve has a Bachelor's degree in Mathematics and holds a Masters degree in Mathematics from Temple University. He has served as the principal investigator on two National Science Foundation projects and has given over 100 workshops on geometry and other topics at local, state, national and international professional meetings.

Join us on February 7, as Nick and Steve share with us how Sketchpad 4 brings its full dynamic power to the study of mathematics: from algebra to calculus.

REMEMBER!! You can earn CPDU credits for attending dinner meetings!

Date: Friday, February 7, 2003
Time: 5:30 p.m. Doors Open
6:00 p.m. Social Hour
7:00 p.m. Dinner and Talk
Place: Fountain Blue Banquets &
Convention Center
2300 Mannheim Rd.
Des Plaines, IL
(847) 298-3636
Cost: Members \$29
Nonmembers \$35

RESERVATION DEADLINE
Monday, Feb. 3, by noon, please!
TO RESERVE:
Call 847-295-1068 or
email bowlerjp1234@msn.com
(Pat Bowler-Johnson)
Day or night, leave a message on machine.



**From Southbound I-294 &
Eastbound I-290:**

Exit at I-190 East to O'Hare; Exit onto
North Mannheim Rd.; Take Mannheim
Rd. North 2.25 miles.

From Northbound I-294:

Exit at West Touhy Ave.; Take Touhy
Ave. to Mannheim Rd.; Turn right on
Mannheim Rd.

Public Transit:

Take the CTA Blue Line to the Rosemont
Bus Terminal; Take Pace Bus #223; Exit
at Touhy Ave. & Lee Rd.; Walk East on
Touhy to Mannheim Rd.

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Future Meetings:
March 14 (π day), May 9

Good Things Happen When You Do The Math Right

Richard Rukin, Evanston Township High School

BY JENNIFER JAYSON

Richard Rukin opened his talk on January 17 with several cartoons and jokes that had the whole group nearly rolling on the floor with laughter. The focus of his talk was the idea of rote versus meaningful instruction and what teachers can do to help their students obtain a deeper understanding of mathematics.

Throughout his presentation, Rich showed us several common student errors to which nearly all math teachers can relate. Many of our students make mistakes like these because that they learn tricks to get answers to problems and then apply these tricks incorrectly in different situations.

Here is one example Rich showed the group of how a student had incorrectly solved a problem:

$$\begin{aligned}\sqrt{x+3} + \sqrt{x+5} &= 5 & 2x &= 17 \\ x+3 + x+5 &= 25 & x &= 8\frac{1}{2}\end{aligned}$$

When the student checked the answer, and found out that it did not work, the logical conclusion was that the answer was the empty set. In this particular situation, the teacher had actually taught the student incorrectly, but the process

seemed to make perfect sense to the student—just square *through*—we multiply *through*, divide *through*, so why wouldn't the same trick work with exponents? If teachers stress the separate sides of the equation rather than the operation, perhaps students will have a deeper understanding of mathematical processes.

To show how teachers can stress side rather than operation, Rich showed the following examples:

$$\frac{8x}{2} + \frac{4}{2} = \frac{16}{2} \qquad (3)\frac{1}{3}x + (3)\frac{2}{3} = (3)5$$

Teaching these methods to students can often lead to student confusion because students may start applying the same techniques to exponentiation, roots, trig functions, and other operations. Focusing on the *side* would make the above examples look like this:

$$\frac{(8x+4)}{2} = \frac{(16)}{2} \qquad 3\left(\frac{1}{3}x + \frac{2}{3}\right) = 3(5)$$

Much later in his talk, Rich showed an example of how Pythagorean Theorem problems can sometimes end up looking like this:

$$\begin{aligned}x^2 + 5^2 &= 13^2 \\ x + 5 &= 13\end{aligned}$$

because students want to square root *through* the problem.

Rich also focused on how teaching *key words* can hinder student understanding. Example: Find the supplement of the complement of a 35 degree angle. Student answer: 7975 degrees. How on earth would a student come up with this answer? The word *of* means multiply. So it makes sense to students that the product of the two numbers in the problem is the answer.

Here is another one. Example: Go 5 blocks north then go left 3 blocks. Which way do you have to turn to go

SEE **GOOD THINGS HAPPEN**, PAGE 3

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POINTS AND ANGLES

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Points and Angles is the official publication of the Metropolitan Mathematics Club of Chicago. Founded in 1913, the Metropolitan Mathematics Club is the National Council of Teachers of Mathematics' first affiliate.

Correspondence may be directed to MMC or to the editor:

MMC
c/o Pat Bowler-Johnson
New Trier High School
385 Winnetka Ave.
Winnetka, IL 60093

Isaac Greenspan
Evanston Township High School
1600 Dodge Avenue
Evanston, IL 60204
greenspani@eths.k12.il.us

Good Things Happen

CONTINUED FROM PAGE 2

south? Student answer: 2 blocks. The word *left* means to find the difference.

Last one. Example: A flock of sheep has 125 sheep and 5 sheepdogs. How old is the shepherd? Student Answer: 25. When the numbers are far apart, divide.

A major gap in understanding also arises from the distributive property (multiplication over addition). Teachers often teach polynomial multiplication in the following sequence. First, monomial times a binomial (basic distributive) $2x(3x+8)$. Students are good at this. They know the trick. Next, multiply two binomials. Teachers show students how to FOIL $(x+4)(9x+2)$. This is sometimes combined with a face or the lemon head—another trick that students memorize. Then teachers give a problem like $(2x+3)(x+3x+5)$ and don't understand why students are baffled or claim that the problem cannot be done. Rich made the point that perhaps we should be teaching the last problem first. Are the three problems really different? Let's teach the students multiplication, not tricks that lead them to believe that the third problem cannot be done.

Rich next turned to mnemonic devices, and how they can inhibit long term student understanding. One example is SOHCAHTOA for remembering the trigonometric ratios. A student who has trouble spelling the mnemonic will forever have trouble using it. Rich even mentioned a mnemonic for remembering the spelling of SOHCAHTOA. We should be teaching the math instead of mnemonic devices.

More mistakes Rich has seen students make:

Perhaps this is the MOVE property:

$$x^4 = 6 \text{ is changed to } x = 6^4$$

The distributive property gone awry:

$$\sin(x+y) = \sin x + \sin y$$

Rationalization of the denominator:

$$\frac{4}{\sqrt{\pi}} = \frac{4\sqrt{\pi}}{\pi}$$

Properties of exponents applied to find the inverse:

$$f(x) = 2x \text{ and } f^{-1}(x) = \frac{1}{2x}$$

Many students do not have a deep understanding of the mathematics. It comes back to rote versus understanding. All teachers are good at giving examples, but we must also remember to give non-examples to stress to our students when problems do not work out the way they might expect. Non-examples are just one method we can use to deepen understanding for our students.

To see if his Algebra 1 students understand the Quadratic

Formula, Rich presents the problem: if $xa^2 + ya + b = 0$, state the Quadratic Formula. Clearly, a student would have to understand exactly what each component of the formula means, and where it comes from to get the problem correct.

While on the topic of formulas, Rich emphasized that many times teachers neglect the *if* part of formulas, which is often the most important part for building student understanding of how a formula is applied. This is especially true of the Quadratic Formula. If $a \neq 0$ and $ax^2 + bx + c = 0$,

$$\text{then } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Another topic Rich touched upon was mathematical language and how it can affect student understanding. He pointed out that informal (and some times inaccurate) language leads to student confusion. Informal language is great for introduction, but a connection needs to be made to the more formal language to stress mathematical accuracy to the students. For example, many teachers use the terms and phrases 'negative x ' (instead of 'the opposite of x '), cross-multiply (this soon gets applied to all problems involving fractions), two negatives make a positive (instead of the *product* of two negatives is a positive), and get the words inverse and reciprocal mixed up. No wonder students get confused.

Teachers need to use specific language to facilitate student understanding. To illustrate this point, Rich took the distributive property and demonstrated several ways it is used incorrectly by students. He pointed out that perhaps these mistakes could be circumvented if we talked about the 'distributive property of multiplication over addition' and the 'distributive property of exponentiation over multiplication,' rather than just the 'distributive property.' Teachers should have students investigate when, and for what operations the distributive property works, and when it does not.

Rich also warned of verbally using abbreviations. "See Peas Eat Easy." What? CPCTC—corresponding parts of congruent triangles are congruent. If students actually hear and say the words rather than the abbreviation, the words are more meaningful.

And in conclusion, Richard again stressed the importance of understanding versus rote learning. He reminded us that, "if a student asks how, it is our job to explain why. It is not the teacher's job to cover the material, it is the student's job to uncover the material, because good things happen when you do the math right."

Thank you, Richard, for a fabulous presentation.

Results of MMC Contest No. 18: Interlocking Bounded Figures

The goal of this year's contest, found in the November 2002 *Points and Angles*, was to place the names of 20 bounded plane geometric figures in an interlocking grid, Scrabble®-style, in a rectangle of smallest area. 31 entries were received. Three of these were variants of the same entry and considered as one. Three entries had illegal words or significant spelling errors and were not tabulated.

All entries but three interpreted the rules as the contest writer intended, namely that the grid be comprised only of the 20 names. Under this interpretation of the rules, the winning entry, shown below, was from **Chenghong Huang**, a 10th-grader at Whitney Young H.S. in Chicago. Chenghong fit the words into a 13 by 20 rectangle (area = 260) and will receive \$50. Second was **Adam Colestock**, a teacher at Frances Parker in Chicago, with a 3 by 88 rectangle (area 264), also shown below. He will receive \$30. Third was the 15 by 18 rectangle (area 270) of **Angie Seltzer**, a free-lance technology designer in Glenview, who will get \$20.

Nancy McCarty's Precalculus class and two other entries from Whitney Young H.S. interpreted the rules as allowing other letters to be added to the diagram to form words to connect the 20 required words. This interpretation made it possible to have a rectangle with dimensions 8 by 30, for an area of 240, as shown below.

After quite a bit of deliberation, the judges decided to offer two first-place prizes because this alternate interpretation was not disallowed by the written rules. Thus this class receives \$50.

Five other entrants found rectangles with areas under 300: Vickie Peng (280), Camille Seaberry (288), and Bonifacio Ramos (289), all students at Whitney Young; Julie Klein (294), a 10th-grader at Highland Park H.S.; and John Summerhays (294), the parent of an Evanston Twp. H.S. student. The 16 other entries had areas as follows: 300 (5 entries), 308, 312, 315, 322, 322, 323, 352, 352, 437, 480, and 504.

Area = 260 (Huang):

S	C	I	R	C	L
Q	A	L			
Q	U	A	D	R	I
A	E	D	I		
R	M	I	P		
D	E	C	A	G	O
	I	K	I	T	E
R	H	O	M	B	U
	C				
	O	C	T	A	G
	T	R			
P	A	R	A	L	L
E	E	I	V	P	
N	C	M	A	H	E
T	T	R	I	A	N
A	A	C	X	O	
G	N	R	O	S	E
O	G	N	G	D	
N	L	O			
	E	N	N	E	A

Area = 264 (Colestock):

P	A	R	A	L	L	E	G	R	A	M		S	Q	U	A	R	E		L	I	M	A	C	O	N
	O	C	T	A	G	O						K	I	T	E		L	E	M	N	I	S	C	A	T
Q	U	A	D	R	I	L	A	T	E	R	A	L													

Area = 240 (McCarty's class):

Q	U	A	D	R	I	L	A	T	E	R	A	L		L	I	M	A	C	O	N		C	A	R	D
	R	O	S	E		O	E																		
S	Q	U	A	R	A	L	L	E	G	R	A	M													

We hope that all of you who worked on this contest or used it in your classes found it to be a fun activity. Please address any comments to Zalman Usiskin, University of Chicago, 5835 S. Kimbark, Chicago, IL 60637.

Points from the Interior

BY PAT BOWLER-JOHNSON

Isn't it cold outside? Many of us recall the days when we got up early in the morning—usually very early—to attend the MMC Conference of Workshops. The temperature was in the single digits or below zero and the Chicago winds produced a wind chill factor that was more than twenty degrees colder. Not only was the sky dark when we left our homes, but the coldest day of the year always seemed to take place on the day of the conference. For some of us, the early morning drive was not just long, cold, and dark; it was also filled with a concern or wish, sometimes accompanied by a prayer, that our vehicle would not break down. When the conference first took place, we did not have cell phones, and it would have been an even more chilling feature of the day if the car decided to stall out.

Once we arrived at the MMC conference, however, all those chilling thoughts and worries just disappeared. We exchanged warm greetings as well as stories of our challenging trip to the conference—there was once a speaker whose car wouldn't start, forcing him to take a taxi from a northern suburb to a far-west suburb for a mere \$70! Why would a person go to such a length to make sure that he spoke at a conference? Dedication? Maybe. A concern for others?

Possibly. Dedication and concern were not the only factors, however. The MMC Conference of Workshops has always provided a casual, relaxed setting in which individuals can share ideas, thoughts, and teaching methods, as well as learn techniques to enrich their knowledge of a topic. They may add some change to their classroom by using a calculator, building polyhedra, exploring the world of fractals or simulations, or just sharing stories.

It is my hope that the MMC Conference of Workshops will provide you with some new energy to get you through these dog days. Much thanks goes to Mary Wiltjer and Virginia Highstone for co-chairing the conference, John McConnell for acting as treasurer, Jennifer Jayson and Argo Community High School for hosting the conference. Also, a heartfelt thank you goes to each of you who assists or attends the conference. It will be a great way to rejuvenate our minds during these chilling times in Chicago. If you will not be able to attend the MMC Conference of Workshops, please try to attend the next MMC meeting as we explore Sketchpad 4 through the eyes of its developers. I look forward to seeing you.

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Evanston, IL 60202

*Make check payable to **MMC**.*

Please use a different form for each person.

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Address _____

Phone _____

School _____

Address _____

Phone _____

E-Mail _____

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Choose one:

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2 year (\$35) _____

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1st year teacher } (\$10) _____

retired }

student }

Donations:

Scholarship Fund _____

Speaker Fund _____

Total amount of check: _____

Check preferred mailing address above.

Change of Address

If you have any questions, e-mail
wiltjerm@eths.k12.il.us

NOTICES & REMINDERS

MMC Scholarship

The Metropolitan Mathematics Club of Chicago is offering \$1,000 in scholarships for high school students who plan a career in the teaching of mathematics. Details on the scholarship, as well as the scholarship application form, can be found on the MMC website:

[HTTP://WWW.MMCCHICAGO.ORG/](http://www.mmcchicago.org/)

Application Deadline: March 17, 2003

CAPS presents a Workshop for Teachers on the Morning of February 8 at the Oak Brook Marriott

Was Pinkerton Right?

Chris Olsen,
George Washington High School,
Cedar Rapids, IA

For more information or to register,
contact Stephanie Casey at SCasey@d113.lake.k12.il.us

If you would like a notice or reminder to appear in Points and Angles, please email the text you would like to appear to greenspani@eths.k12.il.us no later than the date of the MMC meeting preceding the issue in which you would like it to appear.

Speakers Wanted for ICTM 2003

Interested in speaking at ICTM 2003
in Chicago at the Palmer House Hilton?

Contact Bob & Simonette Urbain
at ictm2003@aol.com

THE 2003 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS

All Interested High School Students Welcome

Thursday, February 27, 2003,
4 – 10 pm (with a dinner break)
at Evanston Township High School

Information about the tryout format, the teams,
the coaches, the practices, and the competition,
as well as directions to the tryouts can be found at
<http://www.trica.com/caasmt/>

Your membership renewal date appears in the upper right corner of the label.

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