Points & Angles

Newsletter of the Metropolitan Mathematics Club of Chicago
Volume XLIII No. 6, February 2009

Comparing Models and Possible Solutions of a Paper Roll Problem

Tony Peressini
University of Illinois

By Paul Christmas

From I-90 & Southbound I-294: Exit at I-190 West to O'Hare; Exit onto North Mannheim Rd.; Take Mannheim Rd. North 2.25 miles.

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Friday, February 6, 2009
5:30 PM Doors Open, 6:00 PM Social Hour,
7:00 PM Dinner and Talk

Fountain Blue Banquets & Convention Center
2300 Mannheim Rd., Des Plaines
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$31 for Members, $37 for Nonmembers

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reservations@mmcchicago.org or (847) 486-4690, day or night, leave a voicemail.

Tony Peressini joined the mathematics faculty at the University of Illinois in 1961 and retired in 1998. His teaching interests are in mathematics education, applied mathematics and non-linear programming. He is a co-author of two books for high school mathematics teachers as well as all three editions of *Precalculus and Discrete Matheatics*, a book for high school mathematics students in the University of Chicago School Mathematics Project series.

Tony's wife, Joan, was looking at an ad in a grocery store flyer one morning. There was a special on rolls of paper towels available in two different sized rolls, both 1 foot wide and perforated into sheets 1 foot long. The smaller rolls had an outer radius of about 3 inches and the larger rolls had an outer radius of about 5 inches. Both rolls were wrapped on cardboard cores of the radius 1 inch. The price of the larger rolls was twice that of the smaller rolls. She asked Tony which size was the better buy.

Tony will compare four different solutions based on two different mathematical models for this problem. He'll show us models of the paper rolls as arithmetic spirals and as concentric cylinders. From the spiral model, he'll generate two solutions, one using calculus methods and the other based on some remarkable results in Archimedes treatise on spirals. He will also show two much simpler solutions based on the cylinder model and compare them to the spiral model solutions.

Points from the Interior

By Phil Gartner

With this month's article I have decided to speak about an important issue is mathematics education—the use of Computer Algebra Systems, or CAS. We in the Chicago area are in a leadership position on this front with our having hosted USACAS conferences and with many of the country's leaders in CAS right here in the region. CAS has been around for a long time and has grown in its use over the past years. In a short article and for a newsletter that goes to a knowledgeable group of readers, it is foolish to present a comprehensive statement on the why's and how's of integrating CAS more fully into mathematics curriculum. Rather, let me provide just a few items to consider.

> Now is the time for including CAS more fully in our teaching to improve student learning of high quality mathematics.

NCTM's latest position statement on algebra states that teachers should develop skills in "strategic use of a range of technological tools, including graphing calculators, spreadsheets, statistical software, and **computer algebra systems**" (NCTM, 2008). Numerous research findings (Kieran, for instance) indicate that algebraic skills are being maintained while mathematical understanding of concepts is increased. Mathematics educators around the world are using CAS more and more. Reputable curriculum developers are now including CAS as part of their math curriculum. A few examples include University of Chicago School Mathematics Project (UCSMP), Education Development Center and NSF curricula, Key Curriculum, and Pearson's new CME Project series.

CAS is here to stay. Our response cannot be to deny students access to this powerful learning tool. As with any technological change, we must embrace it and find ways to use it to improve education. This is not to say that there are not viable cases to be made for the use of "no calculator" portions of assessments or for students to show algebraic work where appropriate. We as professionals are to determine when and how the machine is used to maximize pedagogical benefit.

The TI-Nspire CAS calculator is one example of a new calculator that makes CAS accessible in the classroom. The Nspire CAS is allowed on virtually all mathematics tests and contests with the notable

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exception of the ACT. This presents a challenge for schools around Chicago where the ACT is a high-stakes test. However, the ACT should not drive our curricular and technological decisions. Second, pressure for change comes from the high schools that step up and force the issue. Officials from ACT have attended CAS conferences like USACAS and will be studying how the calculator policy and the test may be altered in the future. Practically speaking, though, no one can predict what will happen.

Many of us have perhaps been to sessions on CAS at conferences and need some support. Professional development and the sharing of ideas are key. MMC and other professional organizations will continue to provide such opportunities as we all meet this opportunity head-on. CAS sessions at the Annual Conference as well as CAS-related dinner talks have been an important part of MMC's program for years. We are fortunate to have many of the leaders of the CAS movement in our membership. Now is the time for including CAS more fully in our teaching to improve student learning of high quality mathematics.

Election

The annual election for the MMC Board of Directors is almost here! Information on the cadidates appears on page 7 of this issue of *Points & Angles*. There are three vacancies for board members and one vacancy for president-elect. You will receive your ballot by mail in the next few weeks. Small images of the postcard-sized ballot appear on the next page. Ballots may be brought to the March 13, 2009 meeting or mailed. Mailed ballots must be postmarked no later than March 20, 2009. See your ballot for details.

Sample Ballot

See election information on page 3 and candidate information on page 6.

MMC
c/o Conrad Wayne
1520 Janice Ln.
Munster, IN 46321

Stamp

Address Label

Metropolitan Mathematics Club of Chicago
2009 Board of Directors Election Ballot

Bring this ballot to the March 13, 2009 MMC meeting or mail it in.
Mailed ballots must be postmarked by March 20, 2009. Don't forget the 27¢ stamp!

President-Elect: (vote for one)

- ❑ Donald Porzio — Illinois Math and Science Acad., Aurora

Board Members: (vote for three)

- ❑ Dan Hall — York HS, Elmhurst
- ❑ Sheila Hardin — Oak Park and River Forest HS, Oak Park
- ❑ Janice Krouse — Illinois Math and Science Acad., Aurora
- ❑ Stephanie Levin — Fenton HS, Bensenville
- ❑ Lisa Parker — Lincoln-Way Central HS, New Lenox
- ❑ Jenny Wexler — New Trier HS, Winnetka

Candidate for President-Elect

Donald Porzio
Illinois Mathematics and Science Academy, Aurora
(22 years teaching, member of MMC, ICTM, NCTM, MAA)

During my just-completing 3 year term on the Board of Directors for ICTM, I had the opportunity to influence decisions concerning important (and not-so-important) issues facing mathematics teachers in Illinois. I can think of few better ways of being able to continue in such a role than by presiding over the longest-standing organization for mathematics teacher in the state (except perhaps doing this same job for ICTM, but that will just have to wait).

Candidates for Board Member
(more information in the February 2009 *Points & Angles*)

Dan Hall
York HS, Elmhurst

Sheila Hardin
Oak Park and River Forest HS, Oak Park

Janice Krouse
Illinois Math and Science Acad., Aurora

Stephanie Levin
Fenton HS, Bensenville

Lisa Parker
Lincoln-Way Central HS, New Lenox

Jenny Wexler
New Trier HS, Winnetka

Detach the ballot card below and
bring it to the March 13, 2009 MMC meeting or mail it in.
Mailed ballots must be postmarked by March 20, 2009. Don't forget the 27¢ stamp!

Place Stamp Here

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Getting Math Right from the Beginning

By Carol Nenne

MMC members were given an early holiday present with a fascinating presentation by Angela Andrews. Angela is a specialist in early childhood mathematics education who began her higher mathematical experience by failing Algebra in high school. That experience shaped who she became and how she works with students. Angela described herself as a very open-minded, but opinionated, person. She doesn't apologize for her opinions, which are based on years of reading, study, classroom experience, reflection, personal research and discussion with colleagues.

Angela began by reminding us of our students who say "I don't get it!" to which we reply "What don't you get?" We tend to get frustrated when they reply "All of it!" Angela reminded us that if they don't get it, they probably don't know what they don't get—if they knew what they didn't get, they'd probably get it! Often teachers respond to students who don't "get it" by either giving up ("if you don't know what you don't get, how do you expect me to teach it to you?") or by teaching it again—the same way, using exactly the same words—but slower and louder!

Instead, Angela suggests that we get to the root cause of the problem and get it right—right from the beginning. In her case, she feels that she would have passed algebra except for two major misunderstandings. She didn't understand what the = sign meant, and she didn't correctly understand the order of operations. In some cases, such as her misunderstanding about order of operations where she thought she should multiply, then divide, then add, then subtract (a misunderstanding that many of our students share) she actually got the right answer most of the time, and couldn't understand why she was sometimes wrong. And since she had the right answer most of the time, didn't realize that she had the wrong understanding.

Angela then reminded us of something that we probably know—teachers are very busy! We need to stop and take time to think and reflect. She compared some problems in teaching to those faced by Dennis the Menace, whose character once said "The biggest problem I have is that before I think about what I'm doing, I've done it!" Our homework, she said, was to not think about a lot, but to think very deeply about a few simple things.

So what is one of those few simple things? A common philosophy is that children learn by doing. Angela's opinion is that children do NOT learn by

doing; children learn by THINKING about what they're doing. Action without insight is just a waste of time, whether it's a 3rd grade classroom or a 12th grade classroom. It's the insight and understanding that will help the student long-term. Angela cited a longitudinal study that indicates that an early understanding of math is the best predictor of future success in school. And that understanding comes from more than just memorizing facts or manipulating blocks.

Angela then referenced the research of Robert Wright, et al, regarding foundational skills in numbers. These skills for primary students include knowing the forward and backward sequence of number words, knowing the number that comes before and after any given number, sequencing numbers, ordering numbers, and number recognition and naming. A lack of mastery of these skills is often what is keeping students from succeeding in algebra.

The #1 foundational math skill is counting. There are two kinds of counting—rote counting (1, 2, 3, 4, 5, …) and rational counting (counting objects). Angela then gave us a pop quiz—what is the most foundational type of counting? Rote counting wins here (and most of us missed that!), since you can't rational count if you don't know how to rote count. So then the next question was: what is the most important type of counting? It's still rote counting! Surprised? So were most of the members at the meeting. So why is rote counting the most important? Because rote counting is what allows students to understand the number line. It is the best model of the number system (it has integers, can go to infinity, can show fractions and decimals) and you can carry it with you! You can jump into the number line at any point and count in any direction. Angela's premise is that if students don't get rote counting, they are going to have problems with algebra. If they can't do fractions and decimals, they really can't count forward and backward. She referenced a talk by Lee Stiff, "Arithmetic is all about Counting" where he encouraged teachers to help students learn to first count, then count fast, then count smart. For example, when learning to work with fractions, students should start by counting. Students can start with a fraction, for example 2/3, and then count by thirds or sixths. Her philosophy is that every new mathematics topic should be "counted" first.

Another problem that students have with mathematics involves numeral recognition. Angela referred to studies that say that up to 25% of mathematical errors are due to numeral recognition errors. She told us the story of Andrew, a 1st grade student who came in one day and told her that he could add "big numbers" like 23 and 32. The answer, he said, was 20. Angela's first reaction was to wonder where the number sense was, as the sum was less than both addends. To discover the problem, she asked him to explain his thinking. She reminded us that we should always question students and ask them to explain their thinking, not only when they're wrong, but also when they're right. Andrew's explanation: 23 is like 20 and 3 more, 32 is like 30 and 2 more. So he used "skip-counting" to get 20, 30, 40 and finally 15. Then he added 2 and 3 to get 5 and added the 15 and 5 to get 20. When she wrote down 20, 30, 40, 15, he immediately reacted by telling her it wasn't "that" 15, but the "other one." Confusing 15 and 50, a numeral recognition error, led to an unreasonable answer, even though his mathematical skills involving place value and skip-counting were correct.

We then moved on to stages of early arithmetic. These involve counting objects correctly (emergent), establishing how many are in a set and solving additive tasks by counting all (perceptual), solving additive tasks by counting from 1 (figurative), solving additive tasks by counting on, solving subtractive tasks by counting back, and solving for missing addends by counting up (initial), solving subtractive tasks with the most efficient counting strategy and understanding part/part/whole concepts (intermediate), and using a variety of "non–counting-by-ones" strategies to solve addition and subtraction tasks (facile).

Angela related how many problems come from moving too fast through the stages. Teaching the strategies before students are ready to move beyond counting on can lead to misunderstandings; when students don't understand part/part/whole relationships with whole numbers they will have difficulty moving on to fractions. When students get stuck, they tend to move back to the safest stage, which is generally stage 3—the initial stage. As an example, if students at the high school level are still using their fingers to count, we need to do something about it! The answer is NOT timed tests—frequently that's what caused the problem in the first place. They need to go back to counting on and work on the idea of using 10s. As an example, if students need to do a problem like 47+36, they should think of putting 47 on a number line and then count on by three 10s (57, 67, 77) and then add 6.

See "Angela Andrews" on page 7

Candidates in the 2009 MMC Election

Candidates for Board Member

Dan Hall
York HS, Elmhurst
(8 years teaching, member of MMC, NCTM)

MMC is a great organization that makes a significant contribution to mathematics education, both locally and nationally. I know that it has certainly been the single most significant positive influence on my classroom. I want to continue to support the organization and help to give back to an organization that has provided so much.

Sheila Hardin
Oak Park and River Forest HS, Oak Park
(14 years teaching, member of MMC, ICTM, NCTM)

During the past three years I have enjoyed becoming a part of the MMC organization. The speakers are strong and interesting, providing new information and affirmation about the work we do. I am also impressed with the January conference and the variety of topics that it provides to teachers. I would be proud to be more involved with such a well run, professional organization.

Janice Krouse
Illinois Mathematics and Science Academy, Aurora
(18 years teaching, member of MMC, ICTM, NCTM, NCSM, MDHWCS Board)

My true passion lies in the teaching of mathematics, and helping others become the best teacher they can be. I am constantly working to improve my own practice to benefit not only my students, but also other teachers with whom I work. Promoting MMC and the opportunities it provides to teachers is an excellent means of nurturing new teachers of mathematics across Chicagoland.

Stephanie Levin
Fenton HS, Bensenville
(14 years teaching, member of MMC, NCTM)

Before I began my student teaching, my professor, Roger Day, said that anyone planning on teaching in or near Chicago must join MMC. He explained, "Joining this organization will be the most important thing that you ever do as an educator." He was absolutely correct. Attending the meeting and workshops have been so valuable. I could think of no greater honor than giving back to an organization that has given so much to me and others in the mathematical community.

Lisa Parker
Lincoln-Way Central HS, New Lenox
(14 years teaching, member of MMC, ICTM, NCTM)

Speaker at MMC Conferences, ICTM Annual Meetings, and the NCTM Regional Conference in Chicago. Chairperson of the T^3 Regional Conference in June 2008 in New Lenox, Illinois. Mathletes Coach and volunteer at the ICTM State and Regional Math Contests.

Jenny Wexler
New Trier HS, Winnetka
(16 years teaching, member of MMC, ICTM, NCTM)

I have been glad to have had the opportunity to work with the board for the past several years, and I would be honored to serve the Chicago area math education community for another term.

Candidate for President-Elect

Donald Porzio
Illinois Mathematics and Science Academy, Aurora
(22 years teaching, member of MMC, ICTM, NCTM, MAA)

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Angela Andrews

continued from page 5

To do this, get to the next 10 (you need 3 more to get to 80) and then there's 3 more, so the sum is 83. This idea can also be used for differences and part/whole relationships.

Another issue that Angela brought up was the "American Disease"—place value. When given a value such as 35, students can generally tell that there are 3 tens, but can they tell how many fives or how many ones? Do they think that 35 consists of 5 ones? We generally teach the positional idea of place value instead of thinking about the quantity. We need to worry more about the value, concentrating on the quantity property of place value, not the positional property.

While most of the examples Angela used were from an early elementary curriculum, the ideas presented are certainly applicable to students of any age who are struggling with these concepts. And getting it right—right from the beginning—will certainly lead to many more successful mathematics students in all of our classrooms!

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Fri., Feb. 6	Tony Peressini	**Comparing Solutions of the Paper Roll Problem**
Fri., Mar. 13	Claran Einfeldt	**A Math Cursed Life**
Apr. 18		***MEECAS: Precalculus, Calculus, and CAS***
Fri., May 8	Nick Jackiw	**Using the Newest (Yet-to-be-Released!) Version of Geometer's Sketchpad to Improve Learning**

Send upcoming event items to *ilg@chicagomath.org* no later than the date of the MMC dinner meeting preceding the issue in which the item should appear. All items are subject to editing.

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Your membership renewal date appears in the upper right corner of the label.